Roman Nedela: WL-dimension of CA-graphs

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Weisfeiler-Lehman stabilisation algorithm associates to a graph G of order n a decomposition of the universal relation of rank n into a set of relations $\{R_1, \ldots, R_k\}$ satisfying certain axioms. Each R_i is contained either in G, or in the complement $\bar{G}$, or it is equivalence on the set of vertices. Therefore the set of relations can be viewed as a particular simultaneous colouring of edges of G, $\bar{G}$ and $V(G)$. It follows that $\{R_1, \ldots, R_k\}$ can be viewed as an edge-colouring of the complete graph K_n with a loop added to each vertex. The set $\{R_1, \ldots, R_k\}$ is known in literature under the name *coherent configuration.* If one of $R_i = \{[x, x]\}$ is the complete reflexive relation, the name *association scheme* is used. Coherent configurations associated to graphs share many interesting properties. Perhaps the most important is the following: the automorphism group of the configuration $\chi(X)$ associated to a graph X coincides with $Aut(X)$. This explains the primary motivation of Weisfeiler and Lehman to solve the graph isomorphism problem using the associated configurations. Later, it turned out that the information contained in the coherent configuration is not sufficient to solve the graph isomorphim problem in polynomial time. However, for many classes of graphs (generalised) WL-algorithm gives a complete set of invariants solving the problem in polynomial time. The original WL-algorithm was later generalised to n-dimensional WL-algorithm, where the binary relations R_i are replaced by n-ary relations. For a graph X the WL-dimension is the least n such that the n-dimensional WL-algorithm produces a complete set of invariants. Surprising recent development, explained in a monograph by Grohe, shows that WL-dimension of a class of graphs determines a logic in which the considered class can be defined.

Our aim is first to give a short introduction to the theory of coherent configurations and show how it applies to bound a WL-dimension of circular-arc graphs. Circular-arc graphs are intersection graphs of sets of arcs on a circle. They contain the interval graphs as a proper subset.

On-going joint work with A. Gavryliuk, P. Hell, I. Ponomarenko and P. Zeman